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Antiferromagnetic and bond-order-wave phases in the half-filled 2D optical Su-Schrieffer-Heeger-Hubbard model

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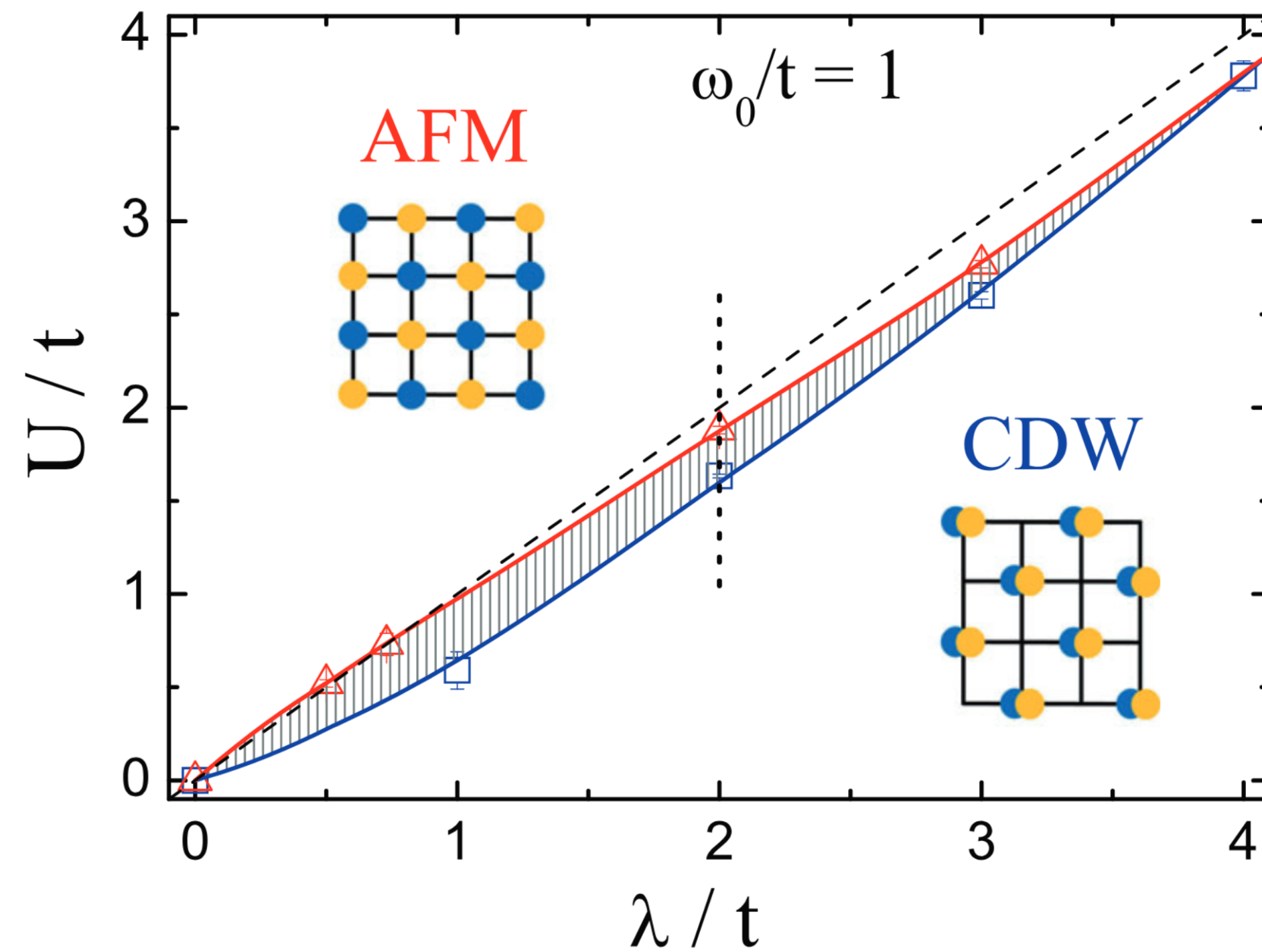
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Electron-phonon models

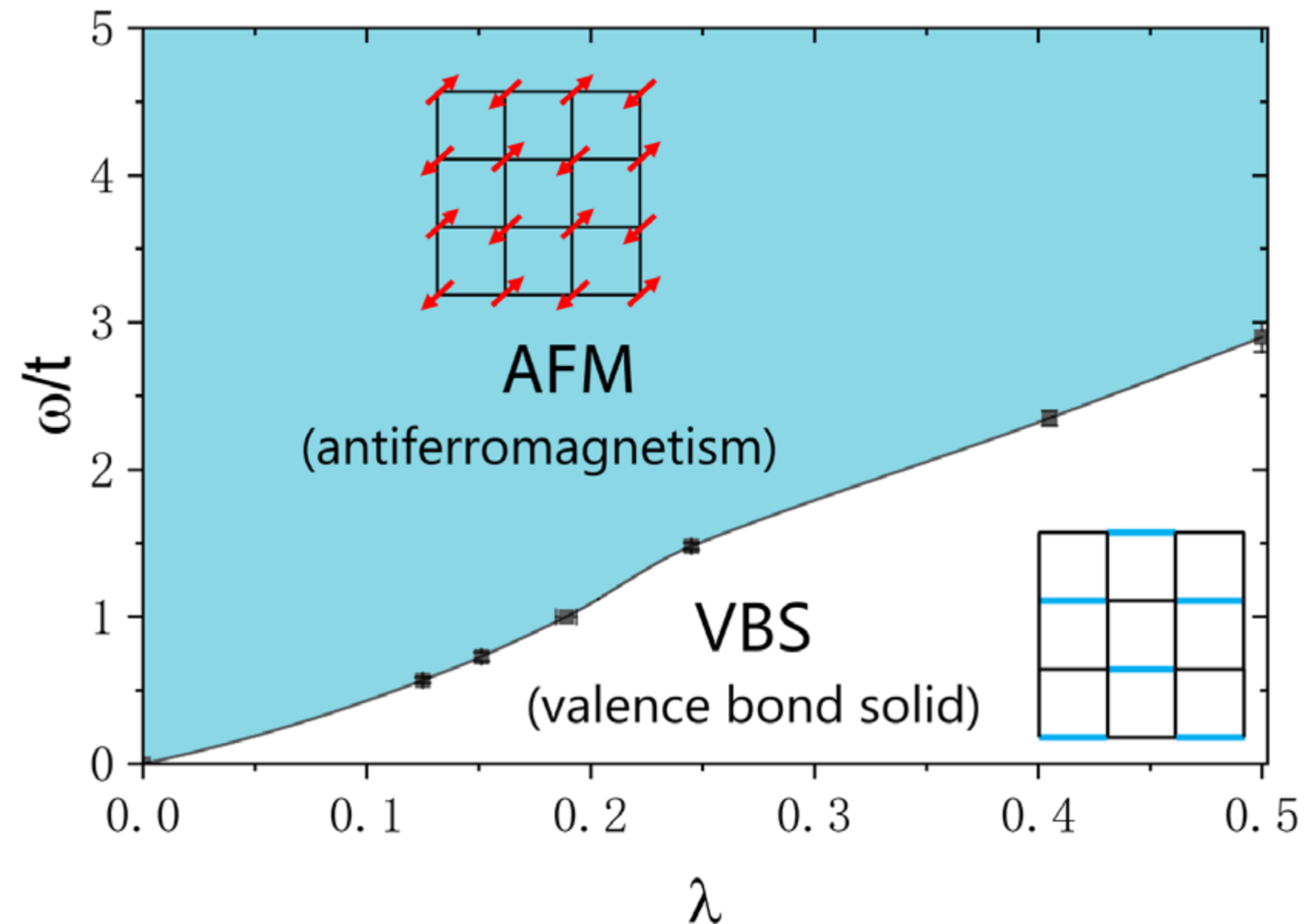
Hubbard-Holstein model



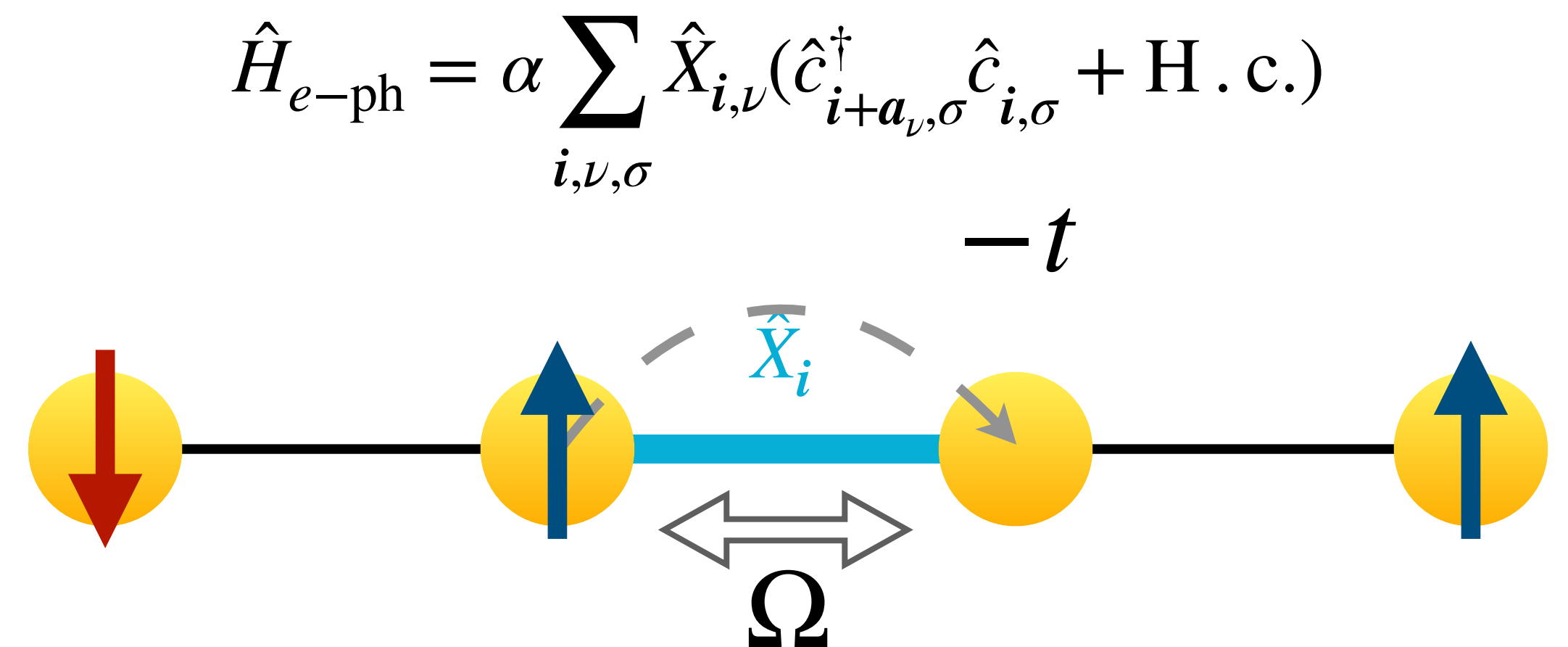
N.C. Costa et al. *Commun. Phys.* **3**, 80 (2020)

Electron-phonon models

Bond Su-Schrieffer-Heeger model

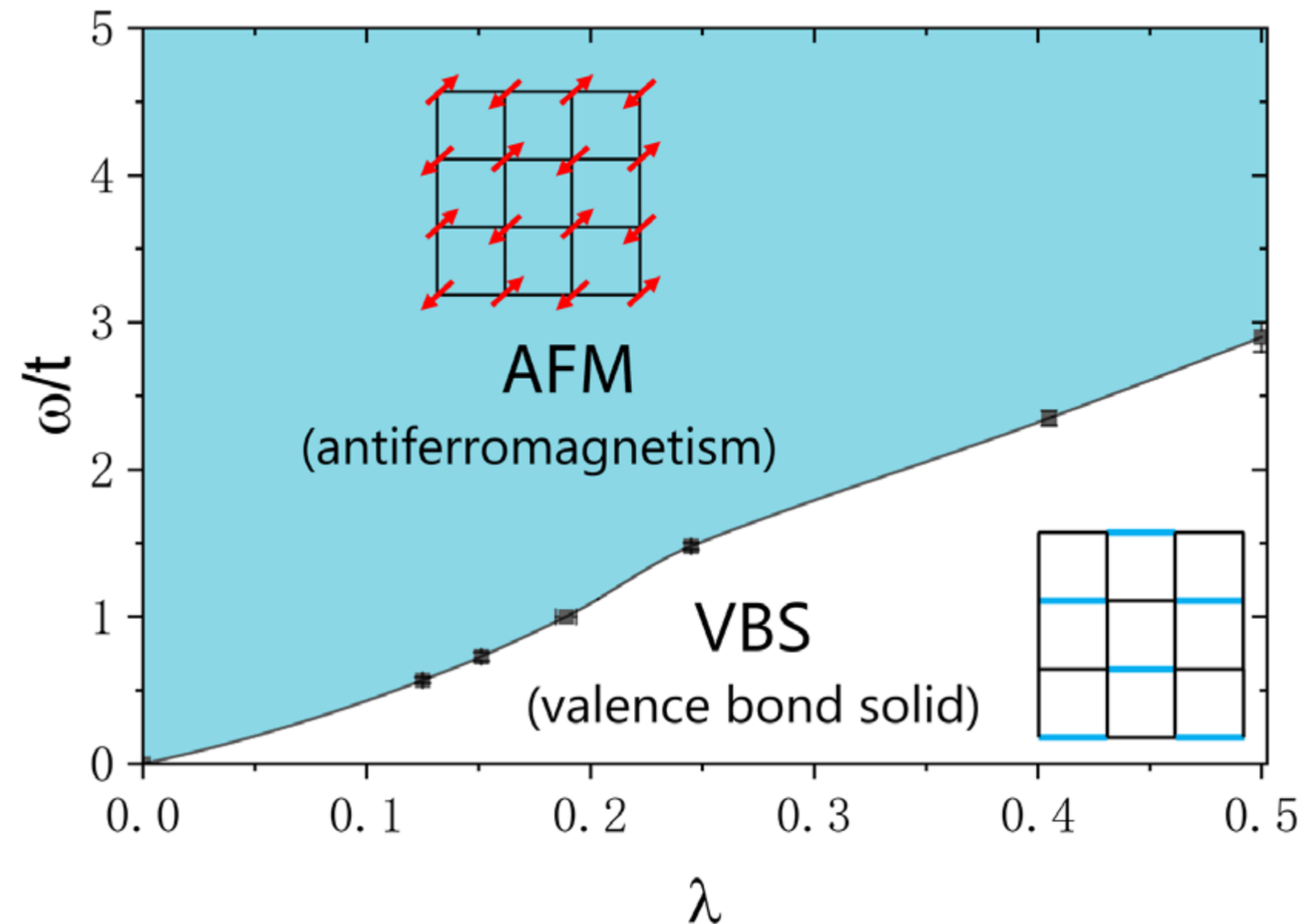


X.Cai et al. *Phys. Rev. Lett.* **127**, 247203 (2021)

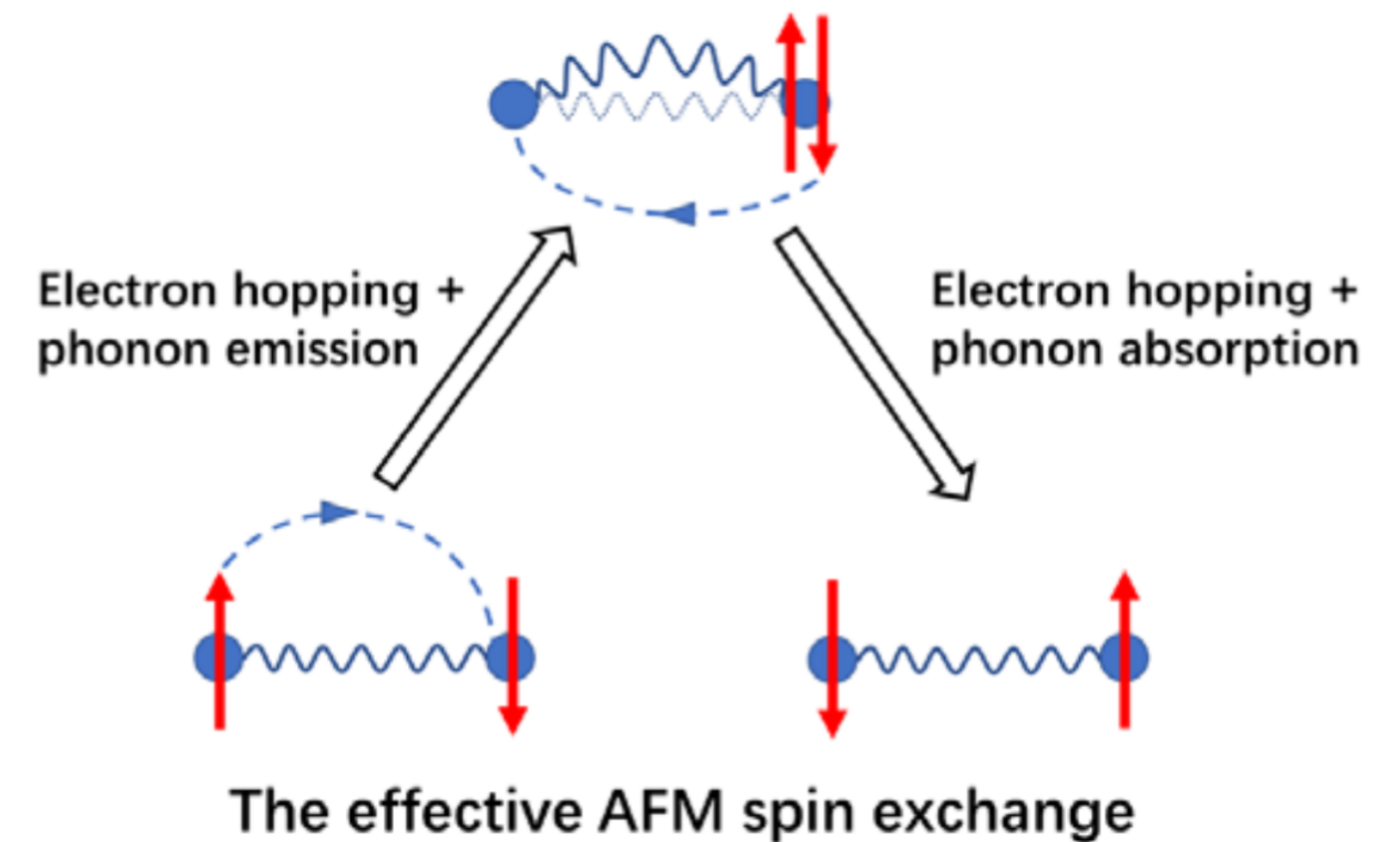


Electron-phonon models

Bond Su-Schrieffer-Heeger model

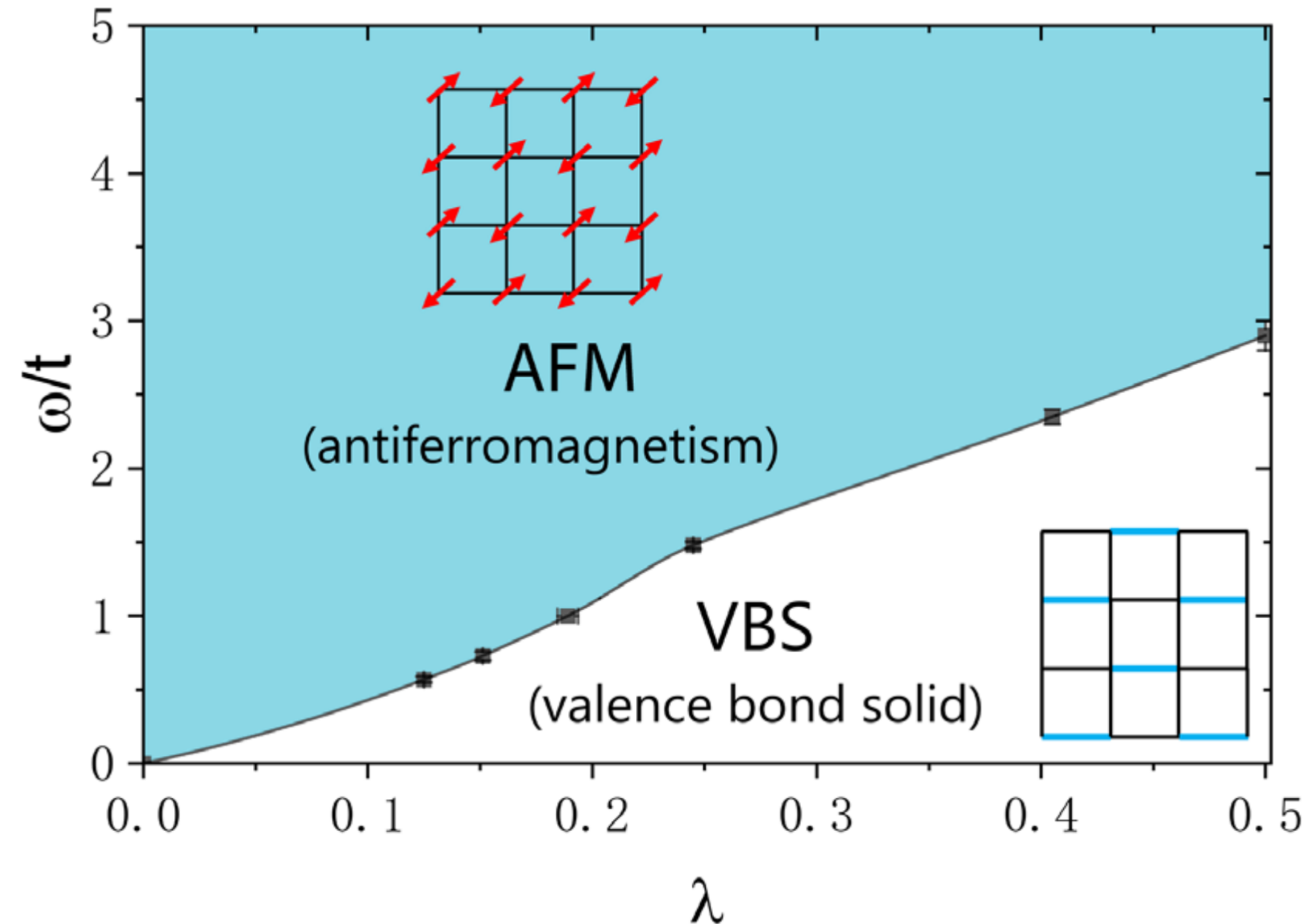


X.Cai et al. *Phys. Rev. Lett.* **127**, 247203 (2021)



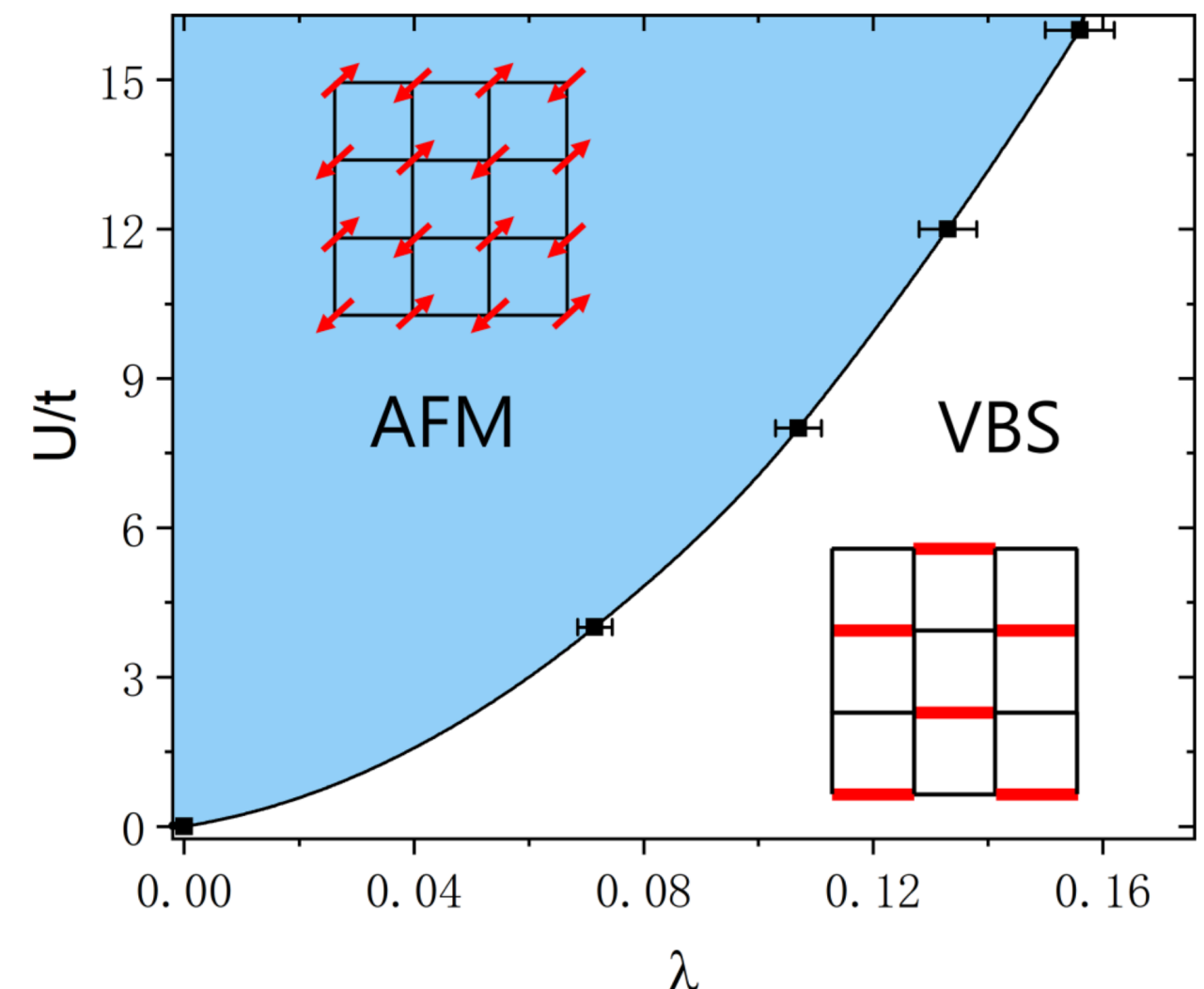
Electron-phonon models

Bond Su-Schrieffer-Heeger model

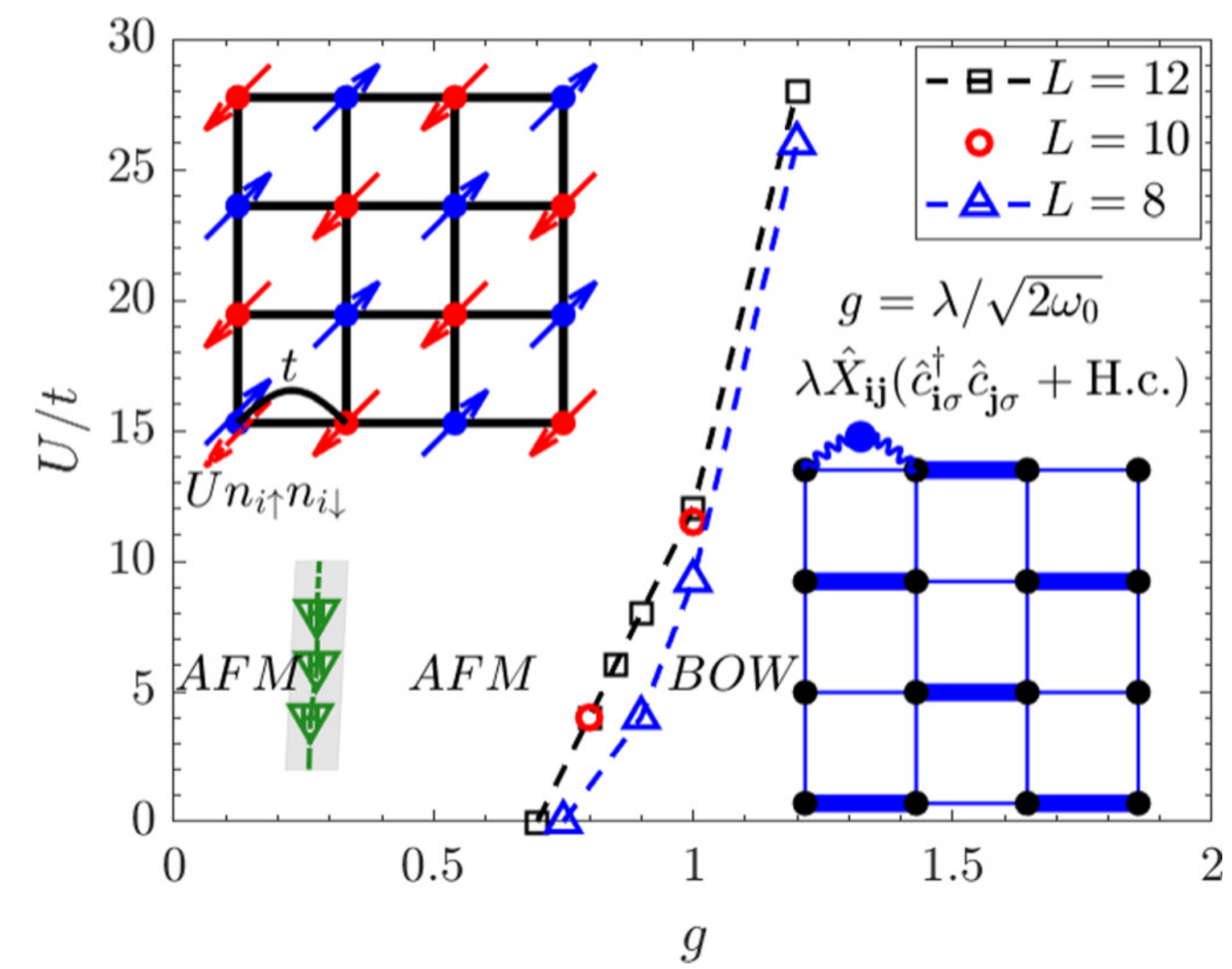


X.Cai et al. *Phys. Rev. Lett.* **127**, 247203 (2021)

+ Hubbard
→



X.Cai et al. *Phys. Rev. B* **106**, L081115 (2022)



C. Feng et al. *Phys. Rev. B* **106**, L081114 (2022)

Optical Su-Schrieffer-Heeger-Hubbard model

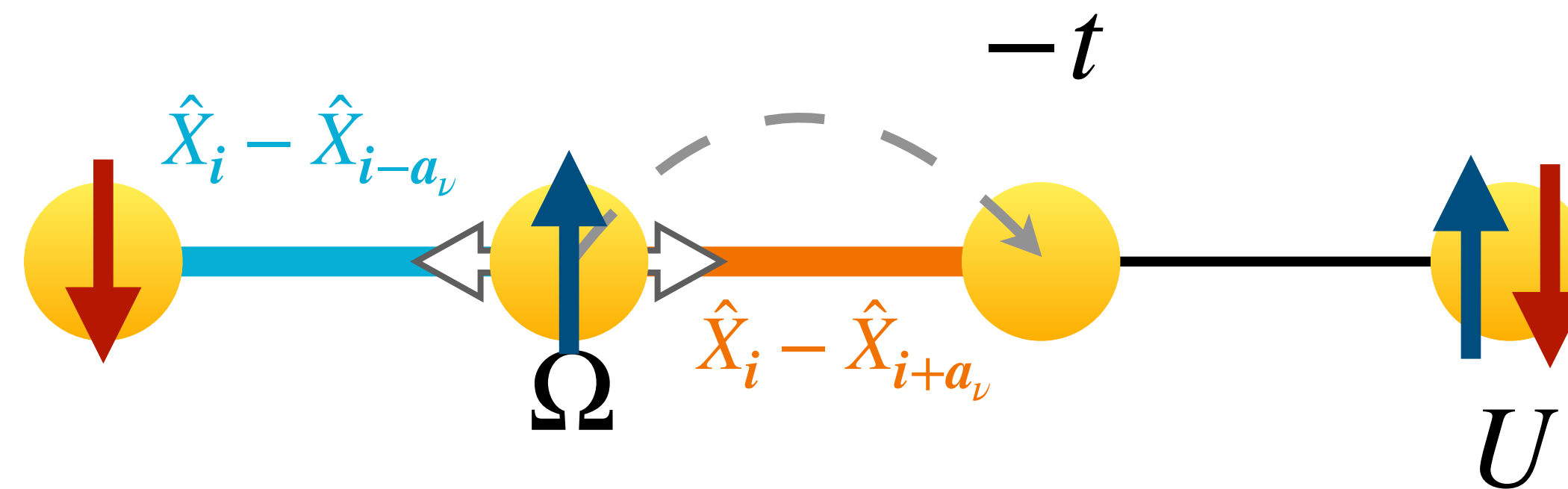
Hubbard model

$$\hat{H}_e = -t \sum_{i,\nu,\sigma} (\hat{c}_{i+a_\nu,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{H.c.}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$

Optical SSH model

$$\hat{H}_{e-\text{ph}} = \alpha \sum_{i,\nu,\sigma} (\hat{X}_{i+a_\nu,\nu} - \hat{X}_{i,\nu}) (\hat{c}_{i+a_\nu,\sigma}^\dagger \hat{c}_{i,\sigma} + \text{H.c.})$$

$$\hat{H}_{\text{ph}} = \sum_{i,\nu} \left(\frac{1}{2M} \hat{P}_{i,\nu}^2 + \frac{M\Omega^2}{2} \hat{X}_{i,\nu}^2 \right)$$



M. Capone, W. Stephen, M Grilli. *Phys. Rev. B.* **56**, 4484 (1997)

Comparative studies: ATL et al. *Phys. Rev. B* **108**, 184501 (2023)

S. Malkaruge Costa et al. *Phys. Rev. B* **108**, 165138 (2023)

Methods

Dimensionless coupling constant

$$\lambda = \frac{2}{WN^2} \sum_{k,q,\nu} \frac{|g_\nu(k,q)|^2}{\Omega(q)} = \boxed{\frac{8\alpha^2}{M\Omega^2 W}}$$

See **ATL** et al. *Phys. Rev. B* **108**, 184501 (2023) and S. Malkaruge Costa et al. *Phys. Rev. B* **108**, 165138 (2023)

Determinant quantum Monte Carlo

- with hybrid Monte Carlo updates
- Finite temperature
- Square clusters ($L = 8 - 14$)
- Phonon energy: $\Omega/t = 0.5$
- Half-filling: $\langle n \rangle = 1$
- **SmoQyDQMC** package



SmoQy Suite: <https://github.com/SmoQySuite>

B. Cohen-Stead et al. *SciPost Physics Codebases*. **24** (2024)

Correlation Ratios

$$R_\gamma(L) = 1 - \frac{S_\gamma(\mathbf{Q} - \delta\mathbf{q}_L, 0)}{S_\gamma(\mathbf{Q}, 0)}$$

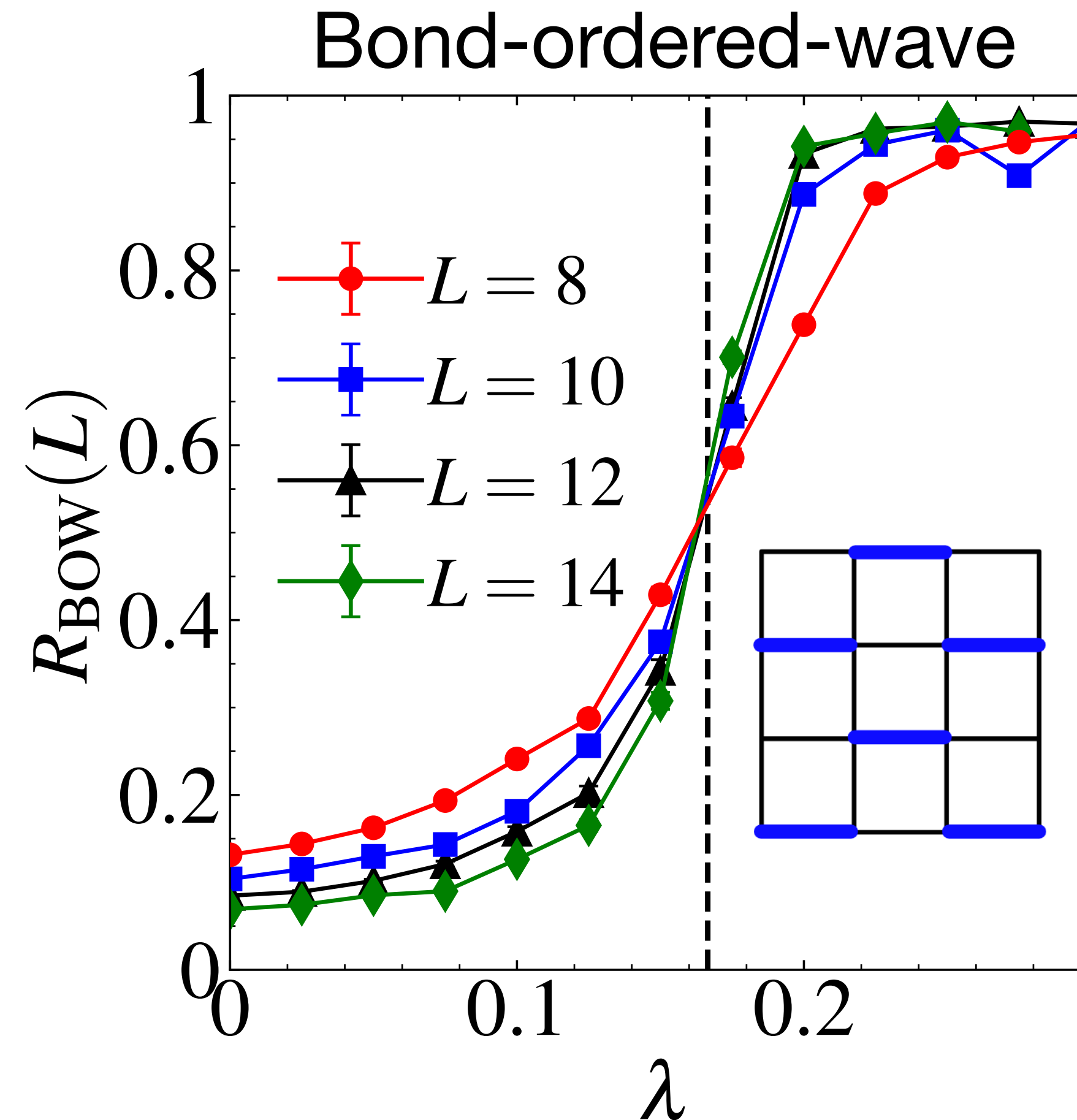
$$|\delta\mathbf{q}_L| = \frac{2\pi}{La}$$

$$\mathbf{Q} = (\pi/a, \pi/a)$$

Finite size scaling
 $\beta t = L$

$$U/t = 3$$

$$\Omega/t = 0.5$$



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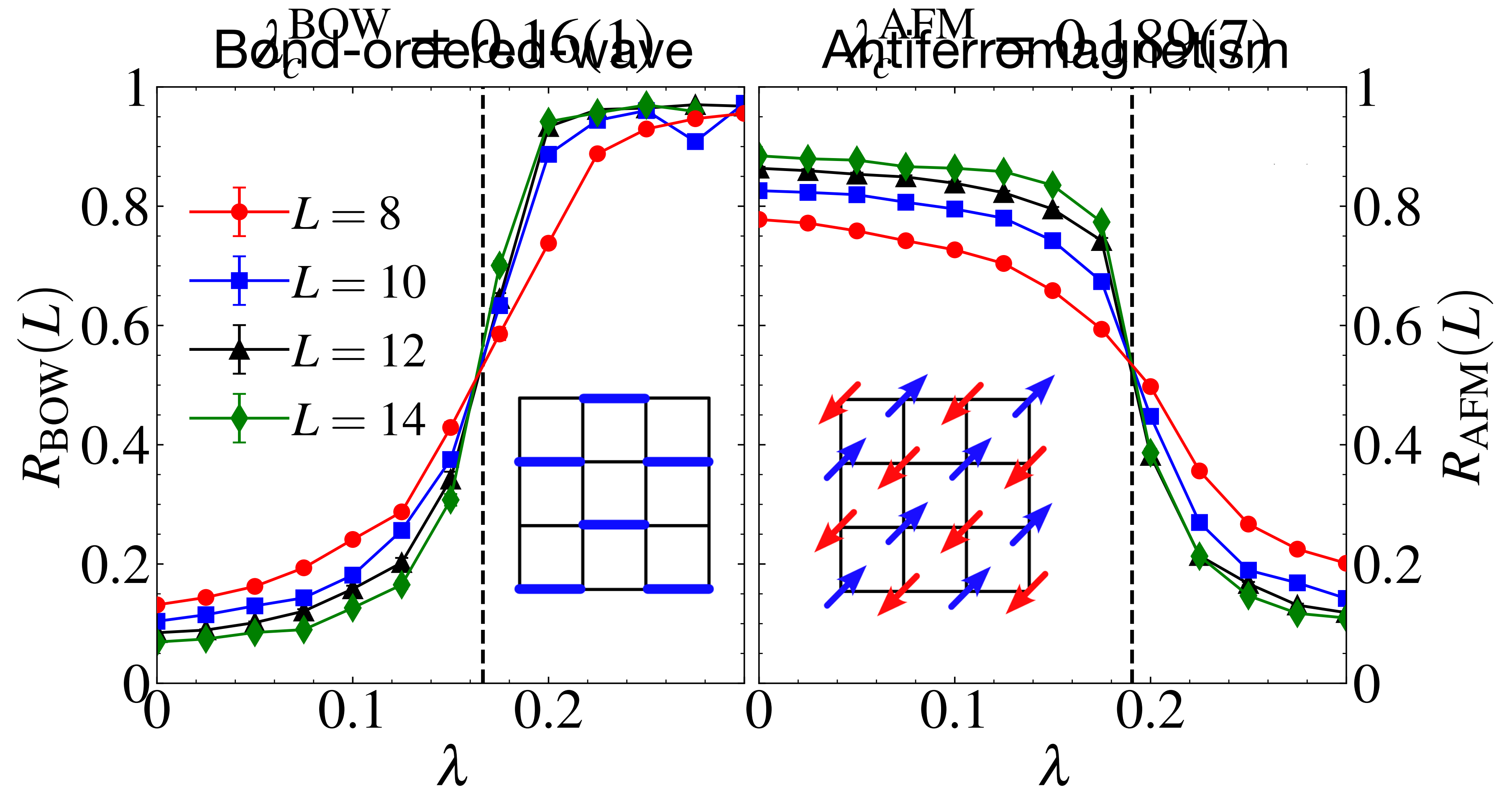
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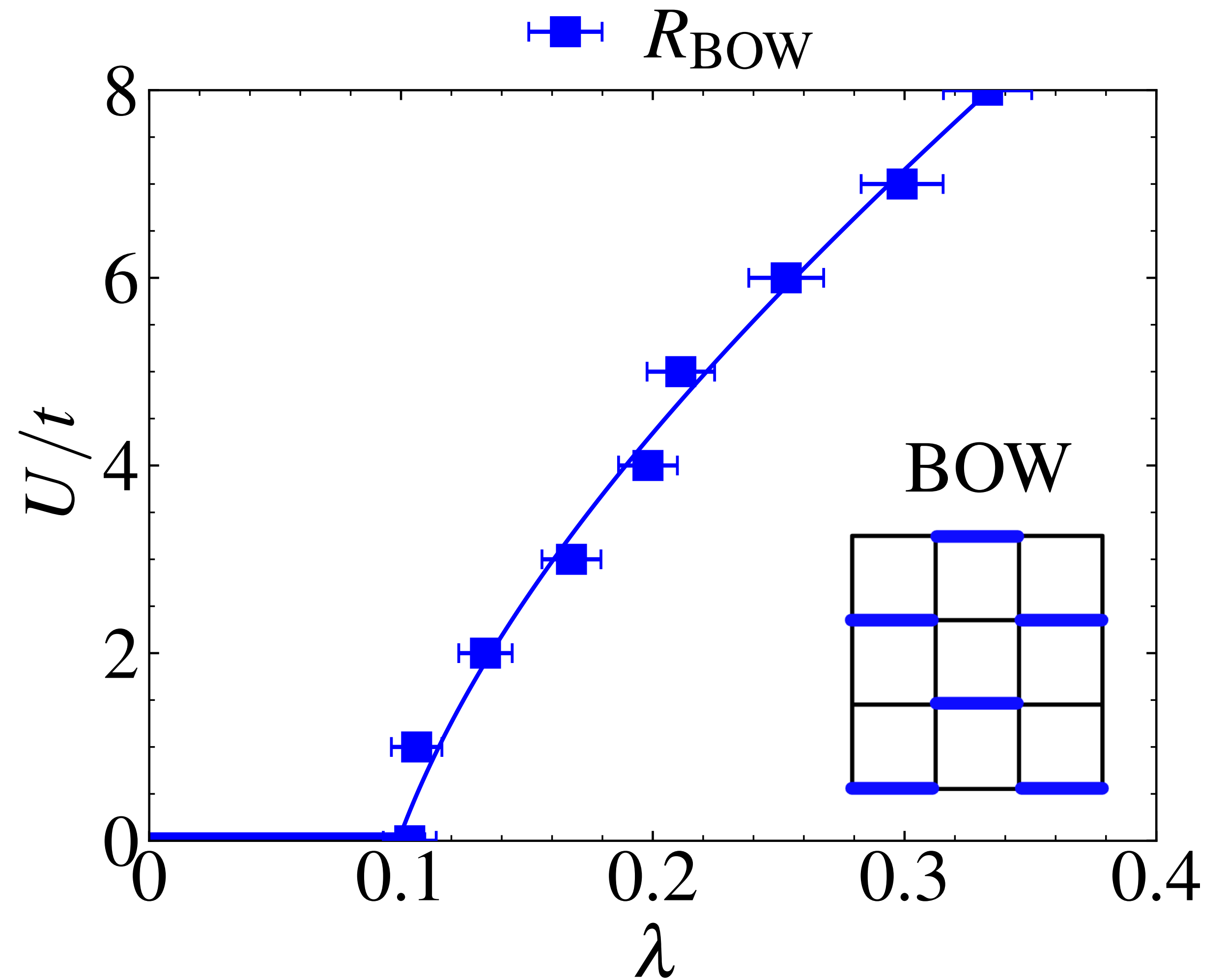
Finite size scaling
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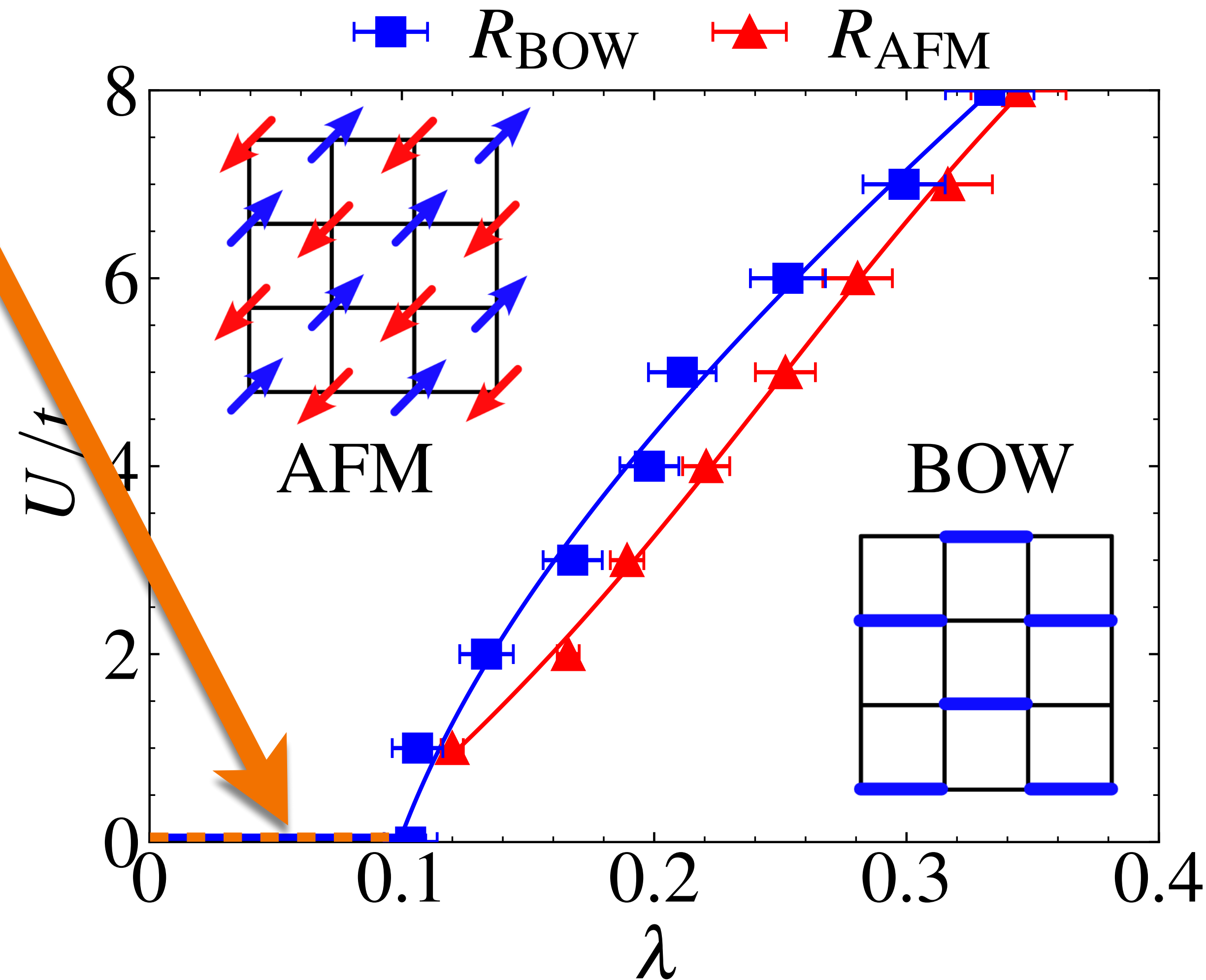
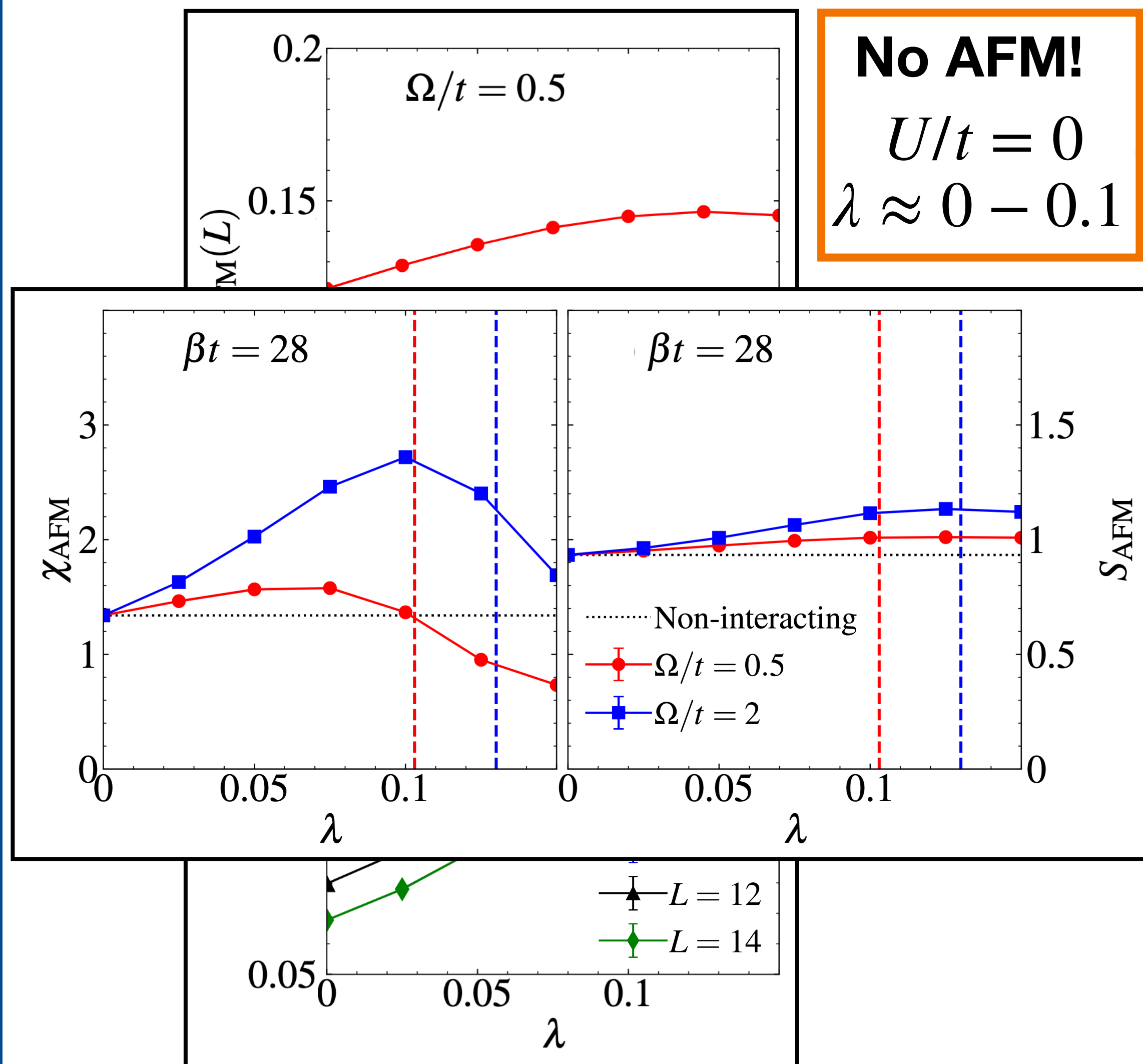
$$\Omega/t = 0.5$$



Phase diagram $\Omega/t = 0.5$

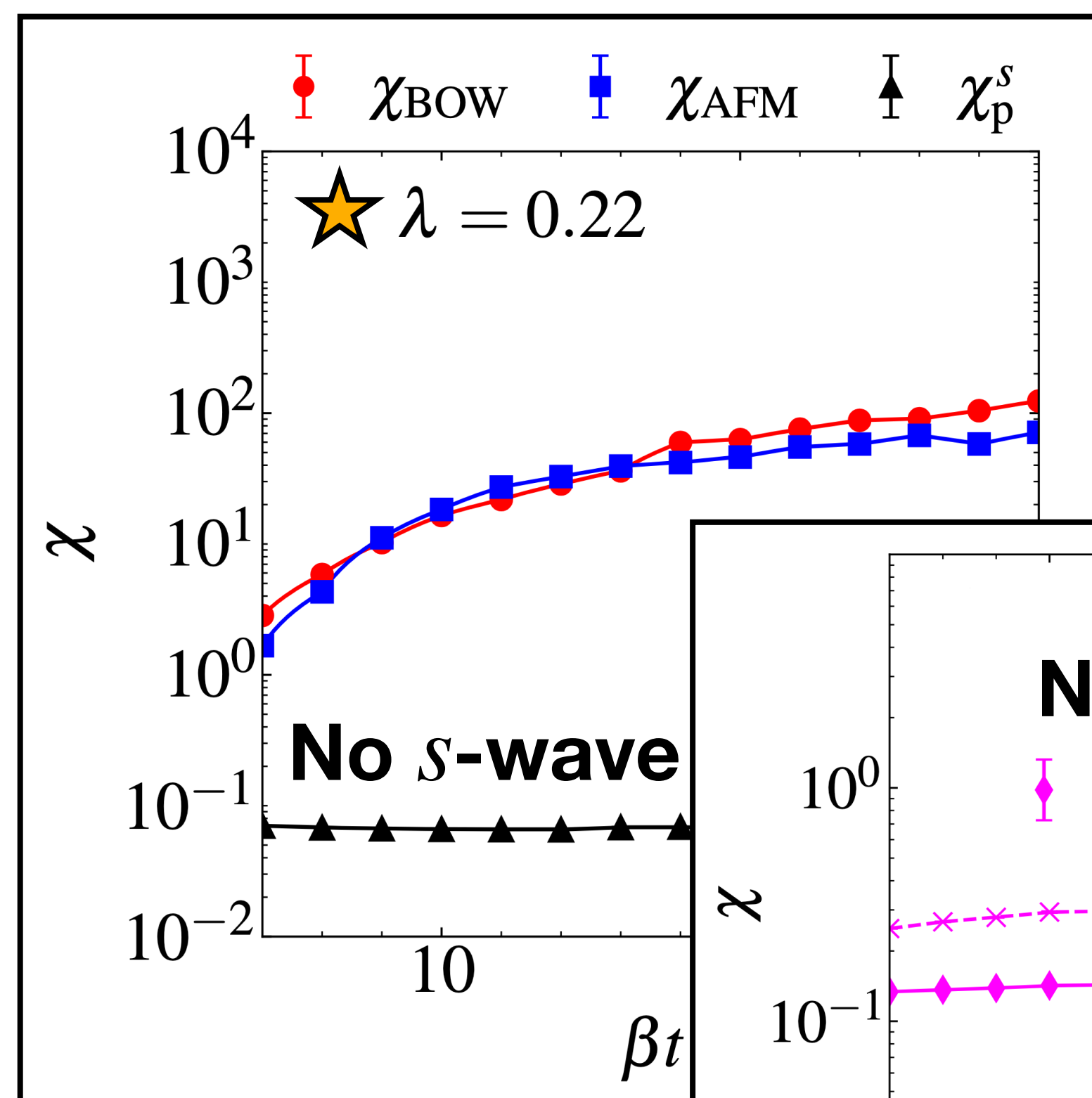


Phase diagram $\Omega/t = 0.5$

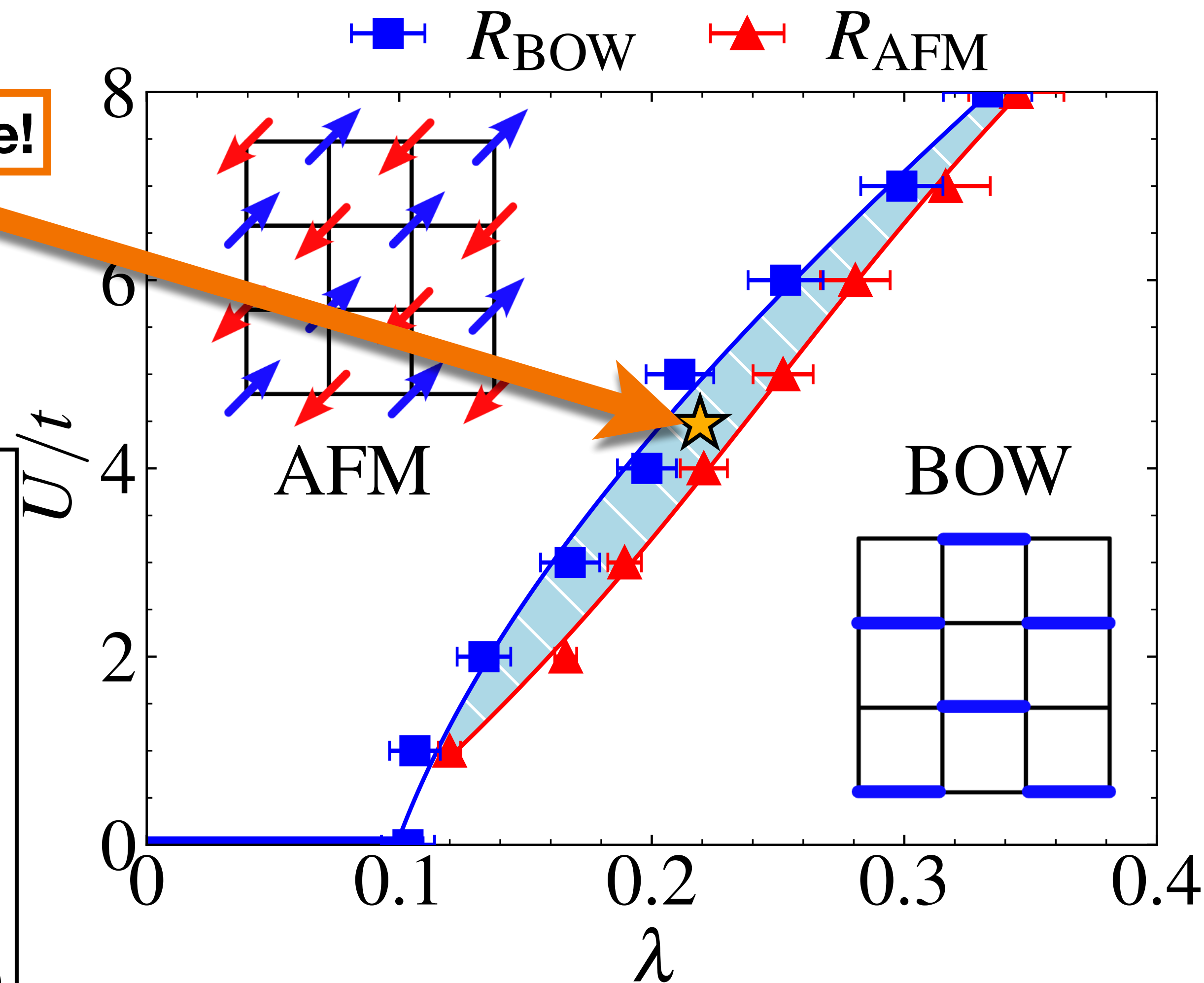
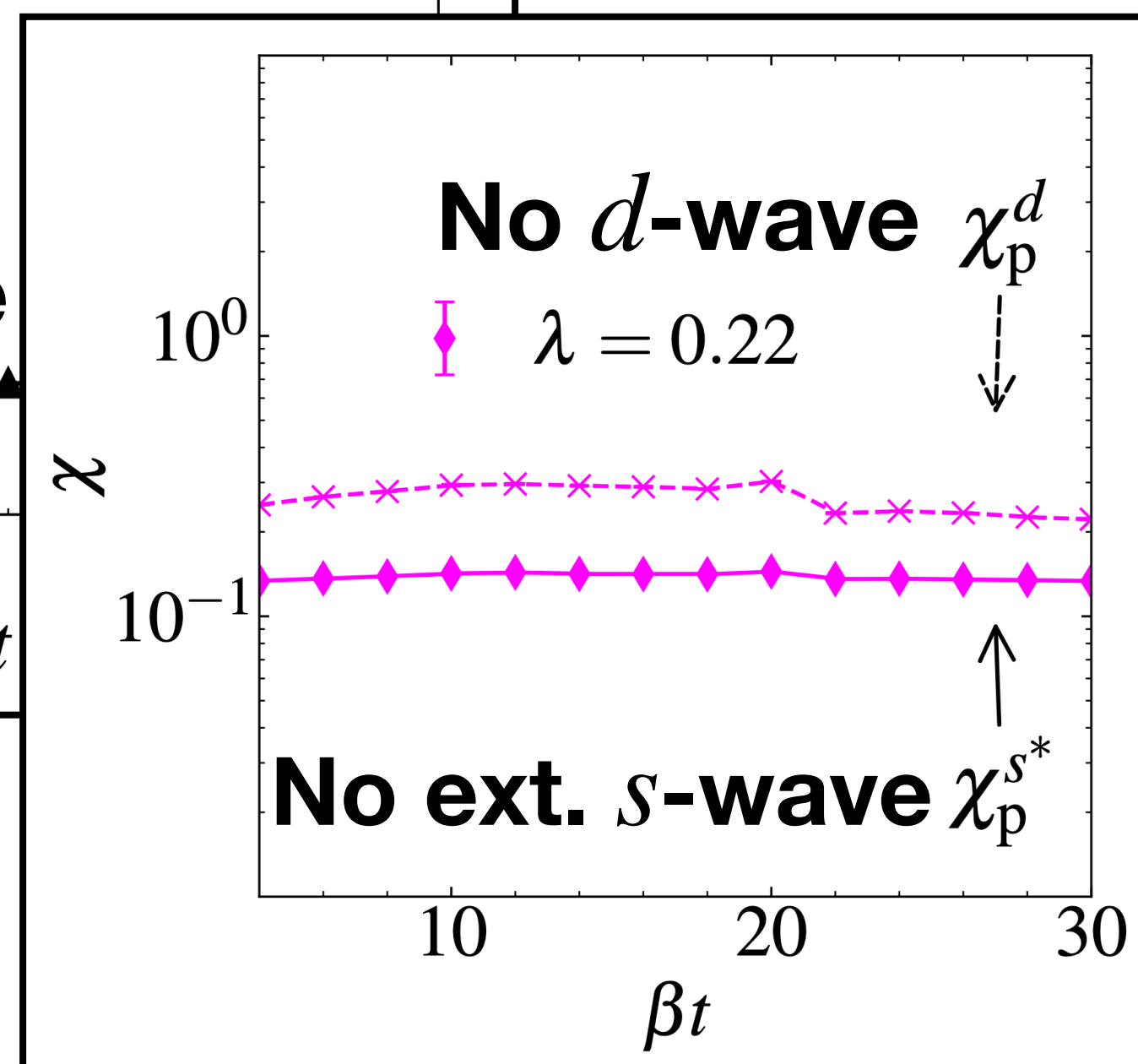


ATL, B. Cohen-Stead, S. Johnston. arXiv:2502.14196 (2025)

Phase diagram $\Omega/t = 0.5$



Coexistence!



ATL, B. Cohen-Stead, S. Johnston. arXiv:2502.14196 (2025)

Summary

- Ground state phase diagram of the optical SSH-Hubbard model.
- No (long-range) antiferromagnetism in the small coupling ($\lambda = 0 - 0.1$) limit at $U/t = 0$.
- Narrow region of coexistence between antiferromagnetic and bond-ordered-wave phases.
- Overall, the pure ($U/t = 0$) optical SSH model behaves quite differently from the pure bond SSH model.
- Preprint available at: **arXiv:2502.14196 (2025)**

